

Electromagnetic properties of resonances

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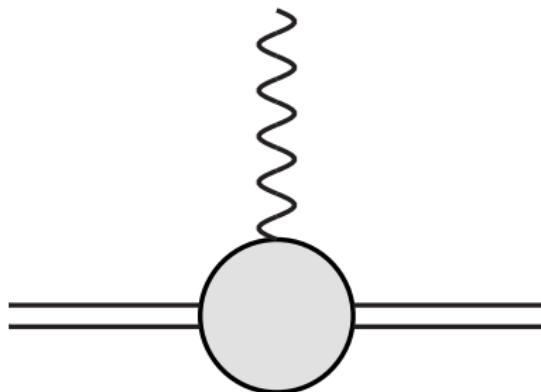
May 2010, Williamsburg Virginia

Outline

- 1 Introduction
- 2 $\Delta(1232)$ E/M Chiral Extrapolation
- 3 $S_{11}(1535)$ E/M Chiral Extrapolation
- 4 W -Boson Magnetic Dipole and Electric Quadrupole Moments
- 5 $S_{11}(1535)$ Energy Shift due to an External Constant Magnetic Field

$\Delta(1232)$ E/M Properties

Single photon interaction



$$\langle \Delta(p') | V^\mu | \Delta(p) \rangle = \langle \Delta(p') | \bar{\Psi} \gamma^\mu \Psi | \Delta(p) \rangle$$

$\Delta(1232)$ E/M Properties

- Decomposition of $\Delta(1232)$ vector current matrix element

$$\langle \Delta(p') | V^\mu | \Delta(p) \rangle = -\bar{u}'_\alpha \left\{ \left[F_1^* \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} F_2^* \right] g^{\alpha\beta} + \left[F_3^* \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} F_4^* \right] \frac{q^\alpha q^\beta}{4M_\Delta^2} \right\} u_\beta$$

- Form factors and moments

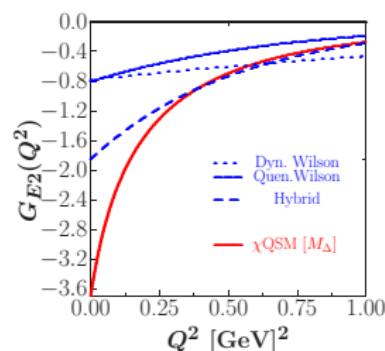
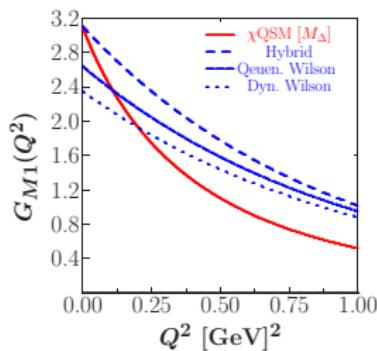
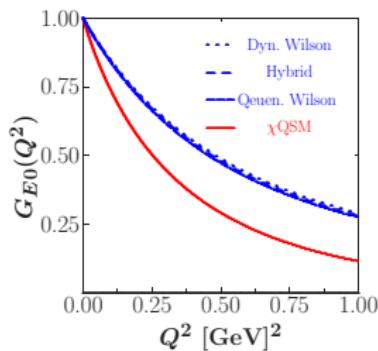
$$G_{E0}(0) = F_1^*(0) \quad e_\Delta = G_{E0}(0)$$

$$G_{M1}(0) = F_1^*(0) + F_2^*(0) \quad \mu = \frac{e}{2M_\Delta} G_{M1}(0)$$

$$G_{E2}(0) = F_1^*(0) - \frac{1}{2} F_3^*(0) \quad Q = \frac{e}{M_\Delta^2} G_{E2}(0)$$

$$G_{M3}(0) = F_1^*(0) + F_2^*(0) - \frac{1}{2} [F_3^*(0) + F_4^*(0)] \quad \mathcal{O} = \frac{e}{2M_\Delta^3} G_{M3}(0)$$

$\Delta(1232)$ E/M χ QSM and Lattice Results



χ QSM

TL, S. Silva, M. Vanderhaeghen 2009

- $\mu_{\Delta^+} = 2.35 \mu_N$
- $G_{E2}(0) = -2.145$

Lattice

C. Alexandrou *et al.* 2009,
 $m_\pi = (691 \sim 353)\text{MeV}$

- $\mu_{\Delta^+} = (1.462 \sim 1.91) \mu_N$
- $G_{E2}(0) = -0.46 \sim -2.06^{+1.27}_{-2.35}$

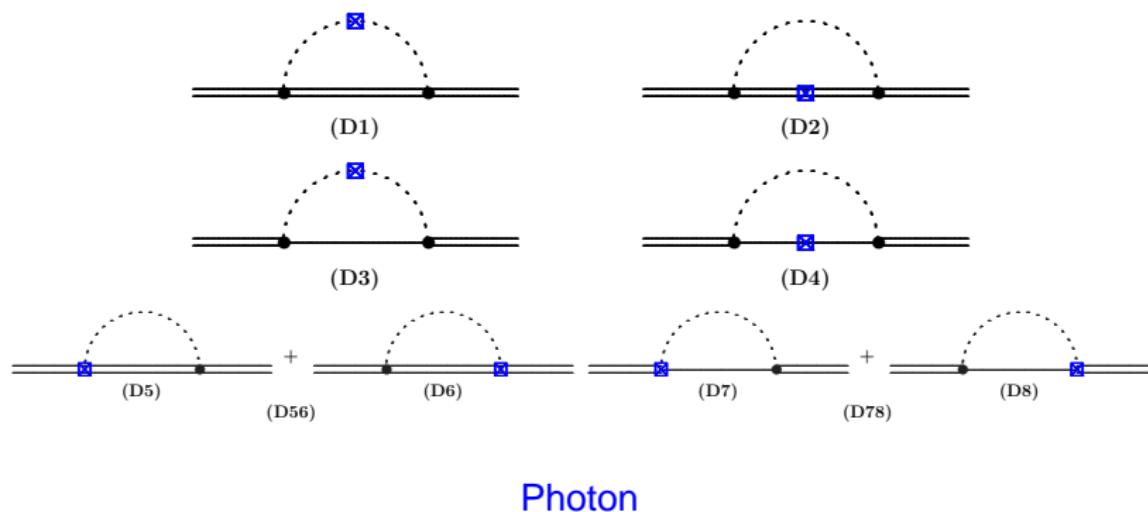
Kottulla *et al.* (2002): Exp. $\mu_\Delta = (2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3) \mu_N$

$\Delta(1232)$

E/M Chiral Extrapolation

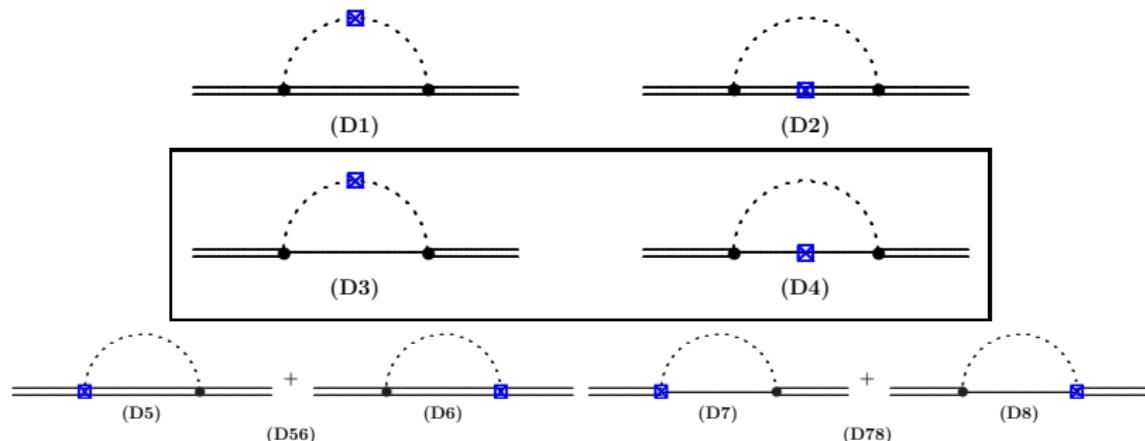
$\Delta(1232)$ E/M Chiral Extrapolation

- Using effective field theory for nucleons, pions and $\Delta(1232)$
- Using *consistent* $\Delta(1232)$ -isobar couplings
of Pascalutsa (1998, 2001) and
 δ power-counting scheme of Pascalutsa and Phillips (2003)



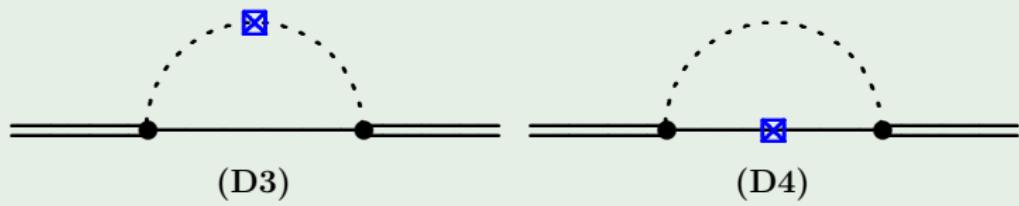
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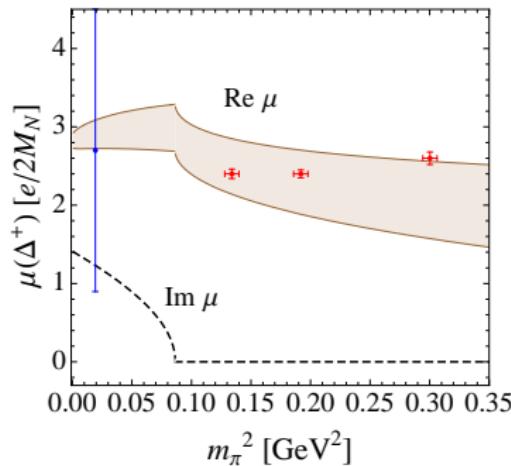
Photon

$\Delta(1232)$ E/M Chiral Extrapolation



$\Delta(1232)$ Chiral Extrapolation of the Magnetic Dipole Moment

Pascalutsa, Vanderhaeghen (2005) $\Delta(1232)$ Magnetic Dipole Moment



Two regions:

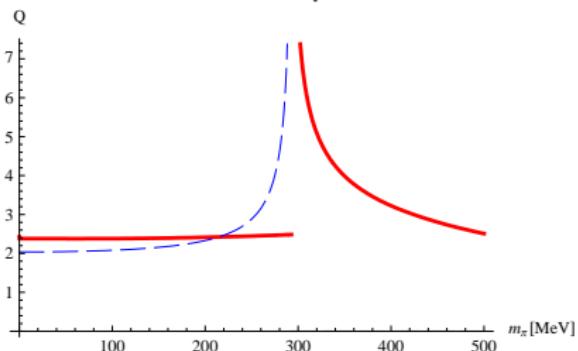
- $m_\pi > M_\Delta - M_N$: $\Delta(1232)$ is stable
- $m_\pi < M_\Delta - M_N$: $\Delta(1232)$ is unstable

$$M_\Delta - M_N \approx 300 \text{ MeV}$$

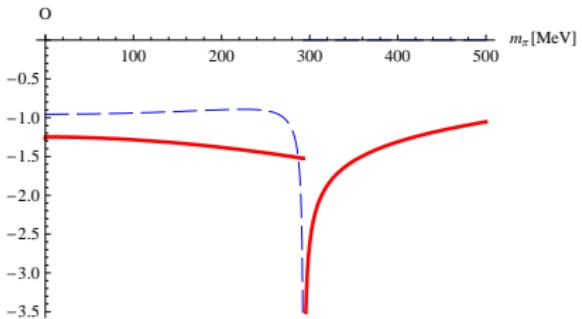
$\Delta(1232)$ Chiral Extrapolations of the Electric Quadrupole and Magnetic Octupole Moments

$\Delta(1232)$

Electric Quadrupole Moment



Magnetic Octupole Moment



Real part, imaginary part

- Singularity at the threshold value

$$m_\pi = M_\Delta - M_N \approx 300\text{MeV}$$

$S_{11}(1535)$

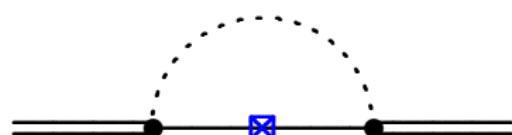
E/M Chiral Extrapolation

$S_{11}(1535)$ A.M.M. via Vertex Corrections

(Toy model)

$$S_{11}(1535): I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$$

$$\mathcal{L}_{NS\pi} = +g\bar{S}\pi^a\tau^a N + g\bar{N}\pi^a\tau^a S$$



Photon

Anomalous Magnetic Moment (A.M.M.) κ

- $\varepsilon_\mu \bar{u}(p')\Gamma^\mu u(p) = \bar{u}(p') \left[iF(q^2) + \frac{\varepsilon \cdot (p' + p)}{2M} G(q^2) \right] u(p)$
- $\kappa = -G(0)$

$S_{11}(1535)$ A.M.M. via Vertex Corrections

$S_{11}(1535)$ A.M.M.

$$\kappa_+ = 2\kappa_1 + \kappa_2$$

$$\kappa_0 = -2\kappa_1 + 2\kappa_2$$

with

$$\kappa_* = -K \int_0^1 dx \frac{[r+1-x]2x(1-x)}{(x-\beta_0)^2 - \lambda_0^2}$$

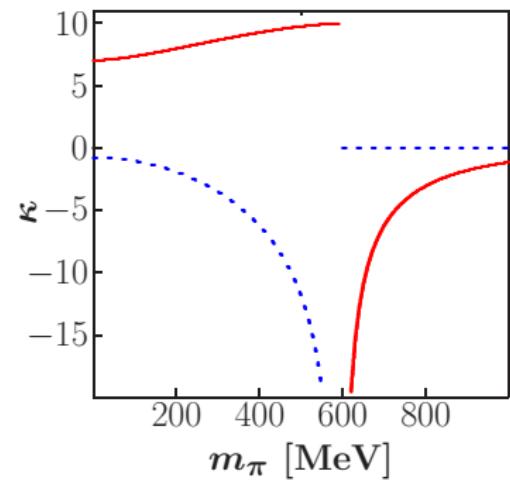
$$\kappa'_* = K \int_0^1 dx \frac{[r+1-x]2x^2}{(x-\beta_0)^2 - \lambda_0^2}$$

Parameters:

$$K = \frac{g^2}{(4\pi)^2}, r = \frac{M_N}{M_*}, \mu = \frac{m_\pi}{M_*},$$

$$\beta_0 = \frac{1}{2} (1 - r^2 + \mu^2), \lambda_0^2 = \beta_0^2 - \mu^2$$

$S_{11}^+(1535)$ A.M.M.



- Imaginary Part, Real Part
- Singularity at $m_\pi = M_* - M_N$ ($\lambda_0 = 0$)

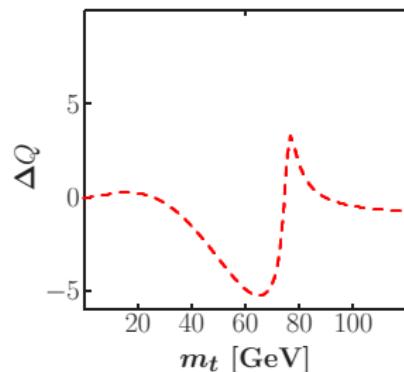
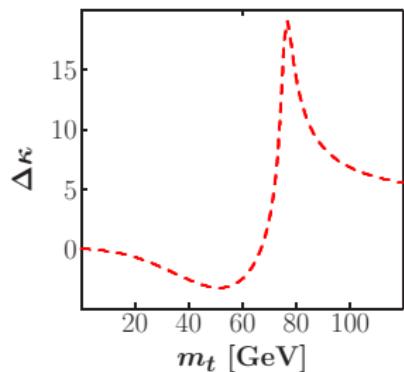
W-Boson

Magnetic Dipole and
Electric Quadrupole Moments

W-Boson A.M.M. and Electric Quadrupole Moment

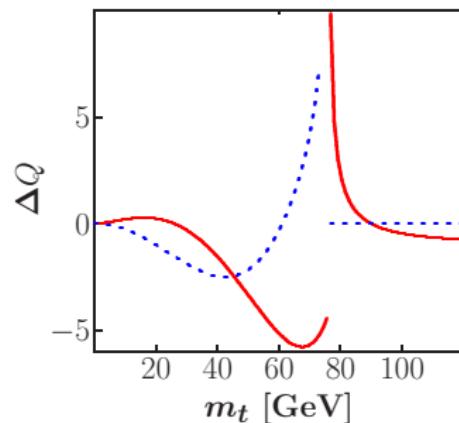
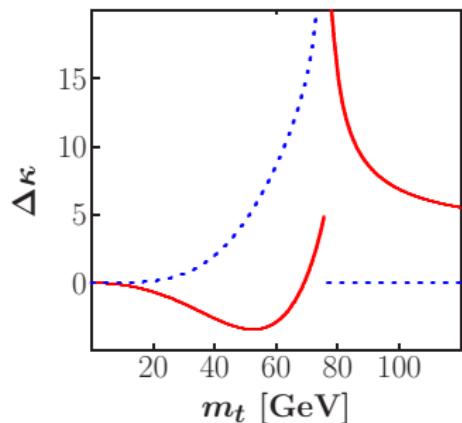
Std. Model: fermion-loop vertex corrections (t- and b-quarks)

G. Couture and
J.N. Ng (1987):



E.N. Argyres *et al.* (1993): "A subtlety arises at $Q^2 = 0$ when the masses [...] satisfy the relation $M_W = m_t \pm m_b$. Although such top masses are not of practical interest [...], it should not pass unnoticed that fermion loops exhibit singularities [...]."

W-Boson A.M.M. and Electric Quadrupole Moment



$$\Delta Q(m_t) = K \int_0^1 dt \left\{ 3 \left[\frac{t^4 - t^3}{A} \right] - 6 \left[\frac{t^4 - t^3}{B} \right] \right\}$$

$$A = t^2 - t (1 + \tilde{m}_t^2 - \tilde{m}_b^2) + \tilde{m}_t^2 - i0^+$$

$$B = t^2 - t (1 + \tilde{m}_b^2 - \tilde{m}_t^2) + \tilde{m}_b^2 - i0^+$$

- To obtain finite result the value of $0^+ = 3 \cdot 10^{-3}$ was assumed in G. Couture and J.N. Ng (1987)
- Singularity at $m_t = M_W - m_b$

$S_{11}(1535)$

Energy Shift due to an
External Constant Magnetic Field

$S_{11}(1535)$ Energy Shift due to an External Constant Magnetic Field

- Energy

$$E(\vec{B}) = M_* - \vec{\mu}_* \cdot \vec{B} + \mathcal{O}(B^2)$$

- Energy shift

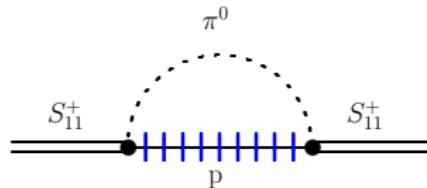
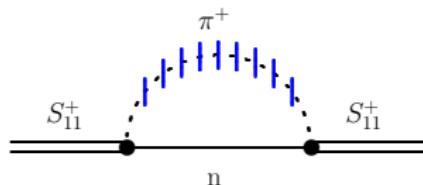
$$\Delta E = -\vec{\mu}_* \cdot \vec{B}$$

- Energy shift due to the **anomalous** magnetic moment κ_*

$$\Delta \tilde{E} = -\frac{\kappa_*}{2} \tilde{B}$$

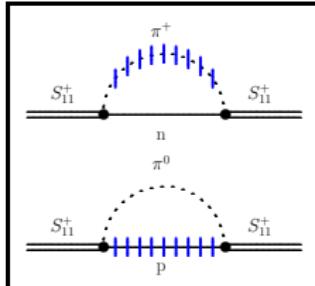
Defined dimensionless with: $\tilde{B} = \frac{eB_z}{M_*^2}$, $\Delta \tilde{E} = \frac{\Delta E}{M_*} + \frac{1}{2}\tilde{B}$, $\vec{\mu} = \frac{eg}{2M_*} \vec{S}$, $g = 2(1 + \kappa_*)$, $\vec{B} = B_z \vec{e}_z$

- Sommerfield (1958): Self-energy in presence of an external field



External magnetic field

$S_{11}(1535)$ Energy Shift due to an External Constant Magnetic Field



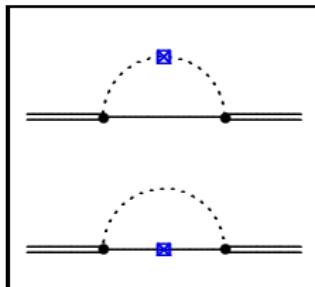
$$\rightarrow \Delta \tilde{E} = \frac{g^2}{(4\pi)^2} \int_0^1 dx (r+x) \ln \left[1 + \frac{x(1-x)\tilde{B}}{x\mu^2 - x(1-x) + (1-x)r^2} \right]$$

$$\rightarrow \Delta \tilde{E}' = \frac{g^2}{(4\pi)^2} \int_0^1 dx (r+x) \ln \left[1 - \frac{(1-x)^2 \tilde{B}}{x\mu^2 - x(1-x) + (1-x)r^2} \right]$$

- Standard def. of magnetic moment (linear B -term in energy)

$$\kappa = -2M_* \frac{d}{dB} \Delta E(B)|_{B=0}$$

- Previous result from vertex correction



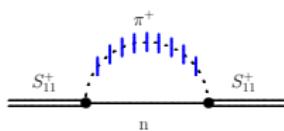
$$\rightarrow \kappa_* = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \frac{-(r+x)x(1-x)}{x\mu^2 - x(1-x) + (1-x)r^2},$$

$$\rightarrow \kappa'_* = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \frac{(r+x)(1-x)^2}{x\mu^2 - x(1-x) + (1-x)r^2}$$

$S_{11}(1535)$ Energy Shift due to an External Constant Magnetic Field

Consider energy shift from first Feynman graph

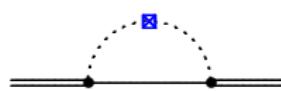
Background field calculation



$$\begin{aligned}\Delta \tilde{E}(m_\pi, \tilde{B}) = & \\ & \frac{g^2}{(4\pi)^2} \left\{ (r + \alpha)(\Omega + \mathcal{A}) \right. \\ & \left. - [(r + \alpha)(\Omega + \mathcal{A})]_{\tilde{B}=0} \right\}\end{aligned}$$

$$\begin{aligned}\Omega &= \lambda \ln \frac{(\alpha + \lambda)(\beta + \lambda)}{(\alpha - \lambda)(\beta - \lambda)} \\ \lambda &= \left[\alpha^2 - \frac{r^2}{1 - \tilde{B}} \right]^{1/2}\end{aligned}$$

Vertex correction calculation



$$\Delta \tilde{E}(m_\pi, \tilde{B}) = -\frac{\kappa_*}{2} \tilde{B}$$

$$\kappa_* = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \frac{-(r+x)x(1-x)}{x\mu^2 - x(1-x) + (1-x)r^2}$$

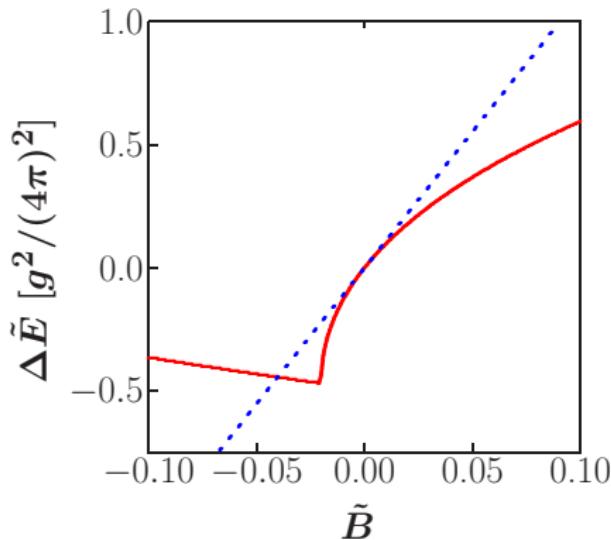
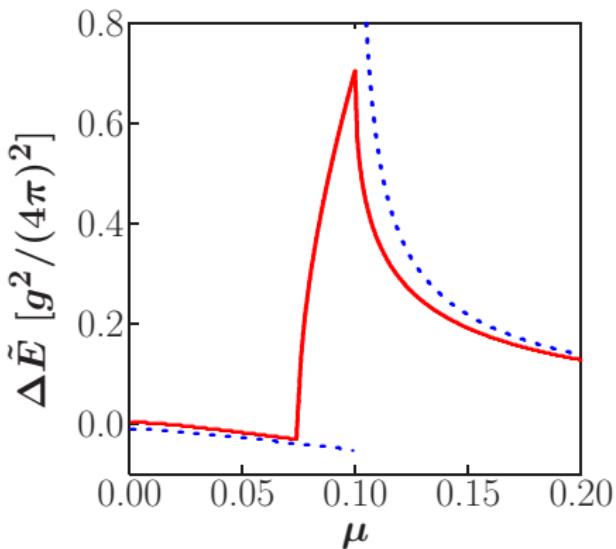
Divergent κ_* \Rightarrow

infinite energy shift by placing
the resonance in an ext. field

$S_{11}(1535)$ Energy Shift due to an External Constant Magnetic Field

TL, V. Pascalutsa, M. Vanderhaeghen
(2010)

$$M_* < m_\pi + M_N \quad (\mu = \frac{m_\pi}{M_*})$$



Vertex correction (one photon exchange): $\Delta\tilde{E}(m_\pi, \tilde{B}) = -\frac{1}{2}\kappa_* \tilde{B}$

Background field: $\Delta\tilde{E}(m_\pi, \tilde{B}) \sim (r + \alpha)(\Omega + \mathcal{A}) - [(r + \alpha)(\Omega + \mathcal{A})]_{\tilde{B}=0}$

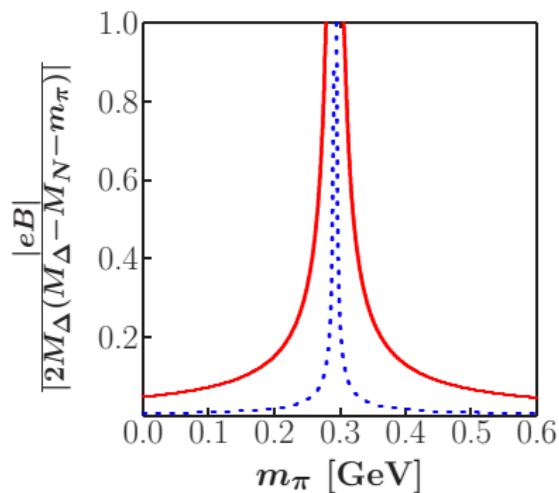
Condition for expanding the SQRT

$$\Delta \tilde{E} \sim (r + \alpha)(\Omega + \mathcal{A}) - [(r + \alpha)(\Omega + \mathcal{A})]_{\tilde{B}=0} \rightarrow \Omega = \lambda \ln \frac{(\alpha + \lambda)(\beta + \lambda)}{(\alpha - \lambda)(\beta - \lambda)}$$

$$\begin{aligned}\lambda &= \frac{1}{2(1 - \tilde{B})} \sqrt{1 - (r - \mu)^2 - \tilde{B}} \\ &\times \sqrt{1 - \left(\frac{M_N}{M_*} + \frac{m_\pi}{M_*} \right)^2 - \frac{eB}{M_*^2}}\end{aligned}$$

Two small parameters

- (1) Weak B -field
- (2) Distance of energy levels



$$\left| \frac{eB}{2M_*} \right| \ll |M_* - (M_N + m_\pi)|$$

Lee *et al.* (2005):

$$|eB| = 0.00108/a^2 \sim 0.00864/a^2$$

Lattice spacing $1/a = 2$ GeV

Condition

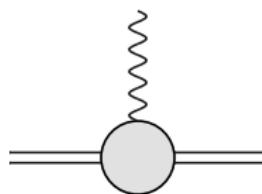
$$E(\vec{B}) = M_* - \vec{\mu}_* \cdot \vec{B} + \mathcal{O}(B^2)$$

$$\left| \frac{eB}{2M_*} \right| \ll |M_* - (M_N + m_\pi)|$$

← interpreting linear energy shift as mag. mom.

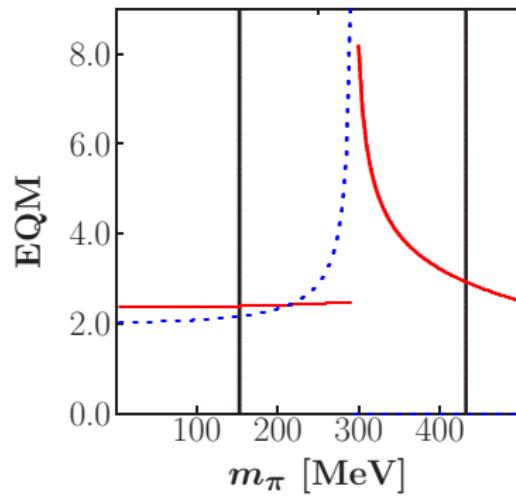
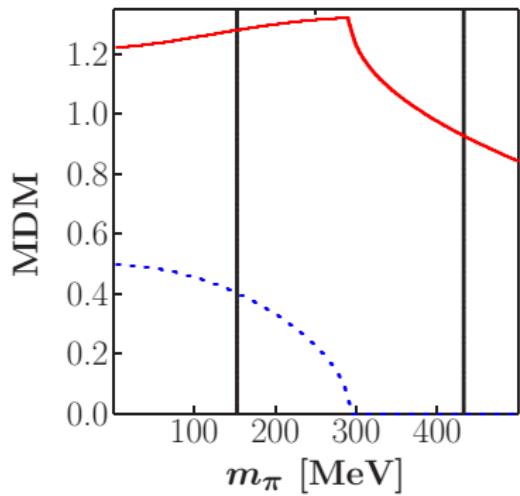
using single photon approximation

→



$\Delta(1232)$ Electromagnetic Moments (Strong B)

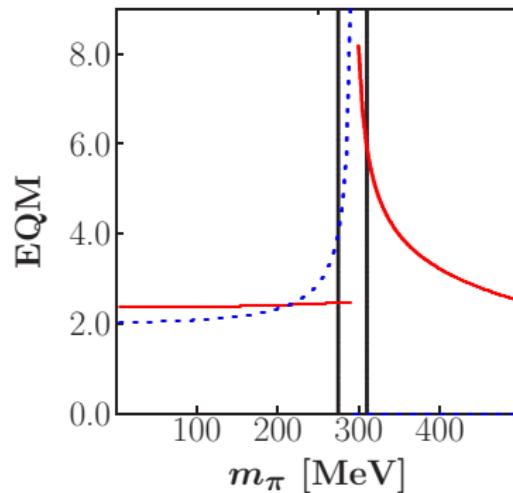
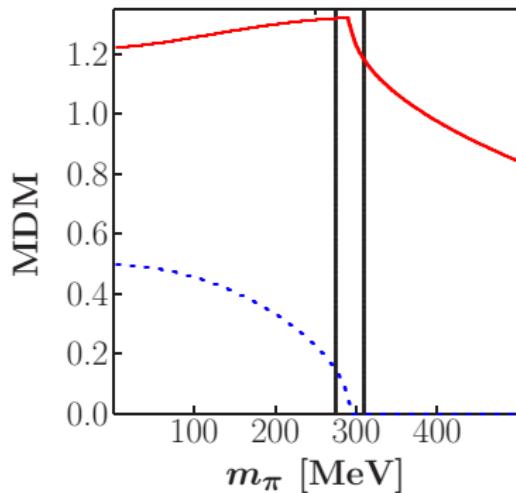
Strong B : $|eB| = 0.00864/a^2$ (lattice spacing $1/a = 2$ GeV)



By considering $\left| \frac{eB}{2M_\Delta} \right| \ll |M_\Delta - (M_N + m_\pi)|$ with $1/10 \ll 1$

$\Delta(1232)$ Electromagnetic Moments (Weak B)

Weak B : $|eB| = 0.00108/a^2$ (lattice spacing $1/a = 2$ GeV)

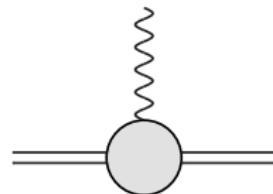


By considering $\left| \frac{eB}{2M_\Delta} \right| \ll |M_\Delta - (M_N + m_\pi)|$ with $1/10 \ll 1$

Summary and Outlook

- Lattice groups start to extract $\Delta(1232)$ E/M properties
- Chiral extrapolation is needed
- Chiral extrapol. by means of E/M vertex cor. exhibit a singularity
- E/M chiral extrapolation of $S_{11}(1535)$ and **W-Boson** electromagnetic moments also show the singularity
- $S_{11}(1535)$ self-energy in presence of an external constant E/M field contains **non-analytic B terms**
- Ansatz of single photon inter. for **resonances** is only valid for

$$\left| \frac{eB}{2M_*} \right| \ll |M_* - (M_N + m_\pi)|$$



- Extension to finite volume corrections for these observables